COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
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M. Bassetti, R. M. Buonanni and M. Placidi : BEAM OPTICS COMPUTATION FOR PARTICLE TRANSPORT SYSTEMS. -

(Nota interna: n. 297)

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1. SUMMARY -

In the present note computation methods for beam transport systems design are exposed. The IBM 7040 code, by which the beam injection system for the Adone storage ring was designed, is also described.

The name of the program is "SYTRAN".

2. INTRODUCTION -

We will consider a beam transport system, consisting of a succession of bending magnets and quadrupole lenses separated by "drift spaces".

The problem consists in the determination of the sets of parameters fulfilling the channel requirements.

For a given system, the same properties can be obtained through several sets of parameters without changing the essential construction: within the several solutions, the one providing the best compromise between practical requirements is chosen.

Each solution is obtained by means of a computer program giving the indications necessary for further selection.

The beam can be represented by points of surfaces limited by closed curves in the horizontal and vertical phase planes.

It is convenient to assume an elliptical shape for such curves because in this case the transformation laws along the channel are simple, and moreover an elliptical envelope is a good picture of reality.

We will describe only what happens in one phase plane with the assumption that the same is true for the other plane.

We will finally assume the particle motions in the two planes to be uncoupled, and will use the "rectangular model" in the mathematical treatment of bending magnets and quadrupole lenses.

3. MATRIX NOTATION AND PHASE PLANE REPRESENTATION -

Let s be the curvilinear coordinate along the optical axis of the system, and y(s) and y'(s) = dy(s)/ds the displacemente and the slope of a particle with respect to the s axis.

The representative ellypse of beam equation is then:

$$W = \gamma y^2 + 2\alpha yy' + \beta y'^2 \qquad (\beta\gamma - \alpha^2 = 1) \tag{1}$$

For our purposes it is useful to describe the centered ellypse (1) by means of the three following parameters:(1, 2)

$$\begin{aligned} R &= a/b && \text{"axis ratio"} \\ X &= c/b && \text{"skew ratio"} \\ W &= ab && \text{"emittance"} \end{aligned} \tag{2}$$

with a, b and c defined in fig. 1.

R and X are related to α, β and γ of (1) through the relations:

$$R = 1/ \qquad X = \mp \alpha/\gamma$$

According to Liouville's theorem, W does not change along the channel.

From (1) and (2) it follows that, for a generic value of s, the maximum values for y(s) and y'(s) are, for a centered ellypse:

$$y^{(s)}_{max} = \sqrt{\beta W} = \sqrt{W(R+X^2/R)}$$

$$y'^{(s)}_{max} = \sqrt{\gamma W} = \sqrt{W/R}$$

To evaluate the effects of the transport system elements upon the phase plane ellypses, the matrix method was used in the computation.

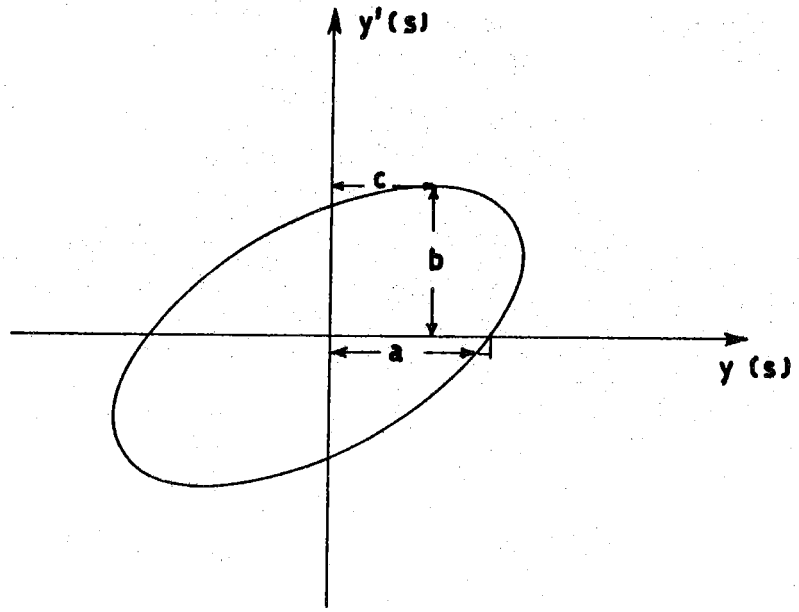


FIG. 1

Let us assume for the generical element a 3 x 3 transfer matrix

$$(4) \qquad M = \begin{vmatrix} a_{11}(s) & a_{12}(s) & a_{13}(s) \\ a_{21}(s) & a_{22}(s) & a_{23}(s) \\ 0 & 0 & 1 \end{vmatrix}$$

The matrix (4) must be applied to the particle representing vector, whose components are y, y', $\Delta p/po$, and whose values at the input and at the output of the element are related by the expression:

$$(4') \qquad \begin{vmatrix} y \\ y' \\ \Delta p/po \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} y_o \\ y'_o \\ \Delta p/po \end{vmatrix}$$

$\Delta p/po$ is the percentual momentum difference of the single particle with respect to the central momentum of the channel.

The elements $a_{ik}(s)$ $(i,k = 1,2)$ modify the ellypse shape without changing the coordinates of the center; the elements a_{13}, a_{23} (which differ from zero only for the bending magnets) yield a rigid displacement of the ellypse.

We shall use for the matrices of the drift spaces, the bending and the quadrupole lenses, the well known expression (for the rectangular model) that can be found in the literature(3, 4, 5).

By means of some purely algebraic calculations it can be shown that, whenever the particle parameters are trasformed according to (4'), the ellypse parameters under-go the following trasformations:

$$R(s) = \left\{ \frac{a_{22}}{Ro} + 2\, a_{21}\, a_{22}\, \frac{Xo}{Ro} + a_{21}^2 \left(Ro + \frac{Xo^2}{Ro}\right) \right\}^{-1}$$

(5)

$$X(s) = R(s) \left\{ \frac{a_{12}\, a_{22}}{Ro} + (1 + 2\, a_{12}\, a_{21})\, \frac{Xo}{Ro} + a_{11}\, a_{21} \left(Ro + \frac{Xo^2}{Ro}\right) \right\}$$

and the ellypse center is displaced by a vector $\vec{\Delta c}$ whose components are:

$$\Delta y = a_{13}\ \Delta p/po$$

(6)

$$\Delta y' = a_{23}\ \Delta p/po$$

so that the relations (3) are, more generally:

$$y^{*}_{max}(s) = y_{max}(s) + |\Delta y|$$

(3')

$$y'^{*}_{max}(s) = y'_{max}(s) + |\Delta y'|$$

4. CALCULATION METHOD AND USE OF THE PROGRAM -

In a transport system design the requirements the channel must meet at its end or at some intermediate section are:

1) - Values of ellypse parameters;
2) - Optical properties such as a focus, an image point or first order achromatism;

3) - Minimum beam envelope;
4) - Maximum resolution in a given section.

From a mathematical point of view, points 1), 2), 3) and 4), yield a set of trascendental equations, for the resolution of which a number of parameters not less than the number of equations is needed.

The parameters we have found the most convenient are the quadrupole lenses strength and the bending magnets field index, for they have both a considerable influence on the optical features of the channel and an extended range of variability.

The solution of the above mentioned non-linear equations system is found by means of a modified Newton-Raphson method which has the following properties:

Let P_o be a real solution of a certain equation system.

Starting from a point P so that $|P-P_o| < \delta$ (where δ is fixed by well defined theorems[6]) the Newton-Raphson method converges to the solution.

If $|P-P_o| > \delta$ the method does not converge.

Our modification allows a wider range for δ value so that if the improved method does not reach a real-solution one can be sure that the initial point P has been chosen in the neighborhood of complex solution. In such a case the computation goes on until the curve defined by the set of the calculated points crosses the surface defined by the vanishing jacobian.

The modification we have introduced to the Newton-Raphson method is explained in the following. Let

$$f_i = 0 \quad (i = 1, \ldots, n)$$

be the system of equations to be solved, P_j the generic non solution point obtained at the j^{th} iteration, and ΔP_j the increment computated with non-modified N.-R. method, by which it should be

$$P_{j+1} = P_j + \Delta P_j$$

We introduce at every iteration, a check function F_j defined by

$$F_j = \sum f_i^2 (P_j)$$

and a reduction factor α, less or equal to 1, so that, taking

$$P_{j+1} = P_j + \alpha \cdot \Delta P_j \qquad [a]$$

it follows

$$F_{j+1} < F_j \qquad [b]$$

In the relation [a] the largest value of α (≤ 1), compatible with relation [b], is chosen.

The quantity F_j vanishes in the case of a real solution, otherwise it tends to a non zero value.

However we are interested only in real solutions and consequently the initial set of the n coordinates of the point P must be chosen following a set of experimented rules we think worthwhile to report:

a) - whichever the arrangement of the quadrupoles succession, that is doublets or triplets, their strengths should be of alternate sign;

b) - the strengths should be chosen so that the focal lenghts be of the same order of the distances between the lenses;

c) - to match four ellypses (two for each phase plane) at the beginning and at the end of the channel length L, it is convenient to start from a lens system having, over the distance in question, an almost unitary transfer matrix;
in this case the quadrupole initial strengths can be easily evaluated in thin lens approximation.

A unitary system, such as the one designed, consists of 2p thin lenses ($p \geq 2$). The effect of a lens doublet and of the corresponding drift spaces is a rotation of each particle by an angle μ in the phase plane; μ verifies the conditions

$$\mu < \pi$$
$$p\mu = q\pi$$

q integer > 0

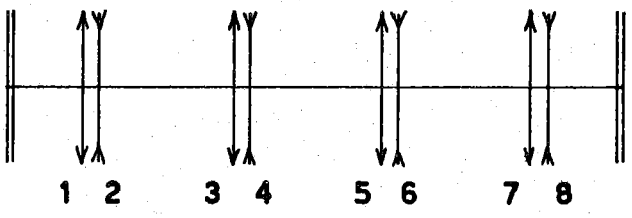
For p=2 only one value of q is allowed (q=1)
For p=3 two values di q are possible (q=1,2)
More generally the number of possible q values is p-1.

The usefulness of a unitary system is due to the fact that it transfers, without changing them (neglecting 2nd order effects), the parameters of the particles and of the ellypses, except for a possible reflection with respect to the origin in the phase plane, over an arbitrary space L; the only limitation on the length L arises from the physical dimensions of the lenses.

On the basis of the above mentioned criteria, it is generally possible to match any four ellypses, increasing the q value of the initial unitary system. In the case one has to change a number of elements greater than 4 (number of equation) only four parameters can be varied a time. It is possible to do so because, as stated above, our program calculates and stores the values of the variables which have decreased the initial quantity $\sum f_i^2$ also when a real solution is lacking. Coming back to the matching, if we have six quadrupoles numbered 1-2-3-4-5-6 we will change previously the

lenses 1-2-3-4 and after, for example 3-4-5-6 ones.

Another way is to fix the values for the strengths of certain lenses: this method is suitable mainly when the lens number is a multiple of four and the geometric structure is a periodic one (see fig. 2);



$g(1) = g(5)$ $g(2) = g(6)$ $g(3) = g(7)$ $g(4) = g(8)$

FIG. 2

The esistence of a real solution for a matching problem is a priori the more probable the greater the q value, but it is not convenient to increase this value beyond the minimum indispensable to get a solution, because the field strengths in the focusing elements and the 2nd order chromatic properties of the transport system increase with this parameter.

5. PROGRAM DESCRIPTION -

The channel optical requirements yeld a set of trascendental equations

(7) $$f_i = o \qquad i = 1, 2, \ldots, j$$

and it is in practice important to know the values of the variable parameters for which the functions f_i are smaller than a prefixed value.

We assume the elaboration is finished when k being an arbitrarily small assigned value, the following relation is verified

(8) $$\sum_{i=1}^{n} f_i^2 \leqslant k$$

Let X_{ij} and Z_{ij} be the horizontal and vertical matrix elements for the part of the system between the initial section and the intermediate one under consideration respectively.

In our program the conditions one may impose (not necessarily all of them) have the following order:

I) $$X_{11} = 0$$

in every system section. Condition I) has the meaning of a focus;

II) $$X_{13} = 0$$

III) $$X_{23} = 0$$

The II) and III) represent first order achromatism conditions for bending magnets (see expressions (6));

IV) $$V - I = 0$$

where V is intended to be the maximum $y^{x}_{max}(s)$ value (as defined by (3')), in both phase planes, and I is a prefixed value, whose choice will be considered later on. Relation IV) represents the condition for beam envelope minimization.

V) $$2X_{12}/X_{13} - D = 0$$

where D is a number proportional to the desired momentum resolution at a fixed beam section.

The I) and V) together realize the required momentum resolution.

Specific values for the total transfer matrix elements TH_{ij}, TV_{ij}, $(i, j = 1,2)$ can be obtained through the following conditions:

VI) $$TH_{11} - O_{11} = 0$$

VII) $$TH_{12} - O_{12} = 0$$

VIII) $$TH_{21} - O_{21} = 0$$

IX) $$TV_{11} - V_{11} = 0$$

X) $$TV_{12} - V_{12} = 0$$

XI) $$TV_{21} - V_{21} = 0$$

Specific values for the final parameters of both phase plane ellypses are reached through the following conditions:

XII) $\quad ROI - ROF = 0$

XIII) $\quad XOI - XOF = 0$

XIV) $\quad RVI - RVF = 0$

XV) $\quad XVI - XVF = 0$

where ROI, XOI, RVI and XVI are the R and X values in the two final planes, evalutated by the computer.

Specific values O_{ij} for chromatic elements of total trasfer matrix can be reached through:

XVI) $\quad TH_{13} - O_{13} = 0$

XVII) $\quad TH_{23} - O_{23} = 0$

Conditions for first order achromatism can be obtained at the same time in two different beam sections by means of II and III, together with the following:

XVIII) $\quad X_{13} = 0$

XIX) $\quad X_{23} = 0$

By means of simple modifications to the program, the choice of different equations is possible.

We shall now examine the program operations in their general lines.

First the following input data are read:

PIPPOT	=	k (from relation 18)
PULSO	=	beam nominal energy (MeV)
E	=	emittance (m·rad)(*)
X, XPR, Z, ZPR	=	ellypse center coordinates in the initial phase planes
RO, XO, RV, XV	=	R and X values in both initial phase planes

(*) - In our computations we can assume for E the same value in horizontal and vertical phase planes, but this is not in general verified.

ROF, XOF, RVF, XVF = R and X values required in both output phase planes

N = total number of transport system elements (drift spaces included)

NUMIT = upper limit for the number of iterations

NONT = upper limit for the number of partial iterations.

N6 = sequence number for the section in which a focus must be realized;

N8, N9 = sequence number for two different beam sections in which first order achromatism is to be obtained (ellypse centers coincident with phase planes origins);

O11, O12, O13, O21, O23, V11, V12, V13 = total horizontal and vertical matrix element values that can be asked for if final ellypses parameters are not assigned;

MM(I), I = 1, . . . , N = indices which describe the succession of the elements in the system with the following conventions:

MM(I) = 1 for a drift space
MM(I) = 2 for a quadrupole lens
MM(I) = 3 for a bending magnet

According to the value of MM(I) the program reads the following data:

For drift spaces:

EL(I), length (meters)

For 4-pole lenses:

ELM(I), equivalent magnetic length in the rectangular model (meters);

G(I) = k^2 . ELM(I) (in thin lens approximation) (meters^{-1}), where k is the elastic constant of the Hill equation describing the particles motion;

For bending magnets:

EN(I), field index

R(I), radius of curvature (meters)

ALF(I), deflection (degrees)

AC1(I) and AC2(I), trigonometric tangents of input and output edge angles

Now instructions corresponding to a set of partial computation in which the total problem is subdivided are given.

For each partial computation the input is:

J number of imposed equations;

N1 initial section of the examined part of the channel;

N2 final section of the intermediate part of the channel;

NS number of the trials to be done

(MUT(I), I=1,...,J), LIST where MUT(I)..... are the labels of the equations to be solved and LIST assumes the values 1 if the envelope must be calculated and 2 on the contrary

KUT(I, 1), I=1,...,J

KUT(I, 2), I=1,...,J

.....

KUT(I, NS), I=1,...,J

labels of the variables to be used.

The program attempts fo find a real solution with the first set of variables; if it goes on,it skips the subsequent sets and comes back to the subsequent partial calculation, otherwise it repeats the trial with the next set of variables taking for the initial values the last ones found in the former trial.

When all the conditions are fulfilled one can attempt to solve the problem of minimizing the beam envelope.

To this aim in our program we adopt a method which although it does not calculate exactily the minimum envelope nevertheless it avoids the use of the Lagrange's multipliers.

The method used is the following:

a further equation, T(4) and a further variable adopted; this equation fixes the maximum value of the envelope along the channel. The initial value that may be assigned is the one the channel has with the last values. Successivelly one tries to decrease this value changing the variables.

If a real solution is found one goes on with the attempt until a non real solution is found, and then takes the last real one.

We shall now give a partial input example.

Suppose we have chosen:

J = 4	N1 = 1	N2 = N	NS = 2
MUT(1) = 12	MUT(2) = 13	MUT(3) = 14	MUT(4) = 15

LIST = 2

KUT(1) = 3 KUT(2) = 2 KUT(3) = 5 KUT(4) = 4

KUT(1) = 1 KUT(2) = 3 KUT(3) = 2 KUT(4) = 4

In this case the choice of equations represents an ellypse matching problem to be solved by the variables 3-2-5-4 in the first try, and by the variables 1-3-2-4 in a successive try; the imposed LIST value means that the print out of the channel envelope is wanted.

If in a next intermediate computation we assume

J = 6 N1 = 1 N2 = N NS = 1

MUT(1)=12 MUT(2)=13 MUT(3)=14 MUT(4)=15 MUT(5)=2 MUT(6)=3

LIST = 2

KUT(1)=3 KUT(2)=2 KUT(3)=5 KUT(4)=4 KUT(5)=6 KUT(6)=7

this means that the program tries to solve also the equations 2 and 3 (1^{st} order achromatism at the $n8^{th}$ section) using also variables 6 and 7.

When the input data are read and printed, the program executes the following instructions: (see block diagram A)

- Starting from the initial section, it calculates the matrices of all the channel elements, employing initial values parameters (point 1 of block diagram A)
- It executes matrix multiplication, obtaining the two transfer matrices from the initial to a general section (point 2 of block diagram A).
- For each of them it calculates the coordinates of the center of the ellypse and the associate R and X values, by applying (4) and (5) to the initial values (point 3 of block diagram A).
- It compares the actual beam envelope values with previous ones and memorizes the largest (point 4 of block diagram A).
- It verifies if the parameters relative to the section must satisfy some equation and, if so, it memorizes their values (point 5 of block diagram A).
- At the final section the assigned equations memorization is complete (point 6 of block diagram A).
- It calculates the quantity (point 11 of block diagram B):

$$FUN3 = \sum f_i^2$$

- It takes initially (point 3 of block diagram B):

$$PIPPO = FUN3$$

and prints its value.

- It calculates the jacobian of the equations system - In order to perform this calculation, it slightly increases each of the parameters in turn (point 5) and calculates again the functions (path 1-2-4-5-1 of the block diagram).

  At the end of the channel it calculates the incremental ratios of the J functions relative to that parameter and treats them as derivatives.

- At this point the program has memorized the values (calculated for the initial values of parameters) of the functions and of their first derivatives and can apply Newton-Raphson's method, to solve the equations.

- It calculates (point 6) the increments to be given to the parameters and the new values for these (point 7).

- It comes back to point(1) substituting new values for the previous ones.

- Afeter point (8) it compares the calculated FUN3 value with the previous one (memorized as PIPPO).

- If FUN3 is less than PIPPO (point 9), it puts PIPPO=FUN3, prints its value and performs a new iteration with new values for the variables; if FUN3 is greater or equal to PIPPO (point 10), it follows again the path (7-1-2-11-8-10-7) with half-values for the increments and examining, at the end, if FUN3 $\gtrless$ PIPPO. If FUN3 $\geq$ PIPPO, path (7-1-2-11-8-10-7) is followed again (always with half of the preceding values of increments) and, if after a number of loops the same result is found, partial elaboration is stopped and the program marks that no real solution is in the neighborhood of the chosen initial set, and goes on varying a new set of variables, or solving another set of equations.
  If FUN3 $<$ PIPPO a new iteration takes place.

After a number of iterations two cases are possible:

a) FUN3 becomes less than k. This means that a real solution (from a practical point of view) has been found.

If the T(4) (or T(5)) equation is among the satisfied equations the program tries to decrease the I (or D) value (see IV or V equations) otherwise it goes on varying another set of equations.

b) When FUN3 does not decrease below the previous value, the computation goes on in the manner described.

The points of the block diagram, where our method differs from the usual Newton-Raphson one are:

1) - the computation of the quantity FUN3 = $\sum f_i^2$ and the comparison of the actual to the precedent value (points 11, 8, 9).

2) - the multiplication of the increments computed by Newton-Raphson method by a factor less than one if the comparison between two subsequent FUN3 values is unsatisfactory.

Besides the general block diagram, we referred to up to now, the one relating the matrix computations more in detail is presented.

Then we want to show an example of a four ellypses matching over a length of 20 meters, the initial values of R and X being assigned, together with the final ones to be realized. The computation is performed using six lenses and it goes through the following steps:

1) - The program tries to solve equations 12, 13, 14 and 15 using the Gs of the lenses 1, 2, 3 and 4, without success.

2) - It tries to solve the same equations with the G of the lenses 2, 3, 4 and 5 with a full success.

3) - It adds the equation 4 to minimize the envelope, and it gets a total failure at the first iteration, by means of the lenses 1, 2, 3, 4 and 5.

4) - It tries to solve these five equations using the lenses 2, 3, 4, 5 and 6; four solutions are found: for each of them the beam envelope is minimized by ang amount of 5%. At this point horizontal and vertical envelopes have about the same value.

5) - The last trial to low the beam envelope is insuccessful.

We want finally to mention briefly the meaning of the quantities given in the output.

G(I) is the variable value, that is quadrupole strength or the field index of the magnet.

CA1,...,CAn, are the values of the constant k in the Hill equation, corresponding to the strengths G(I) and to the magnetic lengths ELM(I).

GRAD1,...,GRADn, are the values, in wb/m^3, of the field gradients in the lenses, corresponding to the lengths ELM(I), for the value PULSO (MeV) of the nominal energy.

The meaning of the numbers under the title "Inviluppo del sistema" Delta energia...., is the following:

DELTA ENERGIA is the energy spread for which the beam envelope is computed.

After this title, the numbers are collected in groups formed by three lines of data; every group corresponds to the output section of the general element. The first two numbers of each fell refers to the radial plane, the two others to the vertical one, and the last number individuates the element in the succession (for the first one, also data the input section are printed).

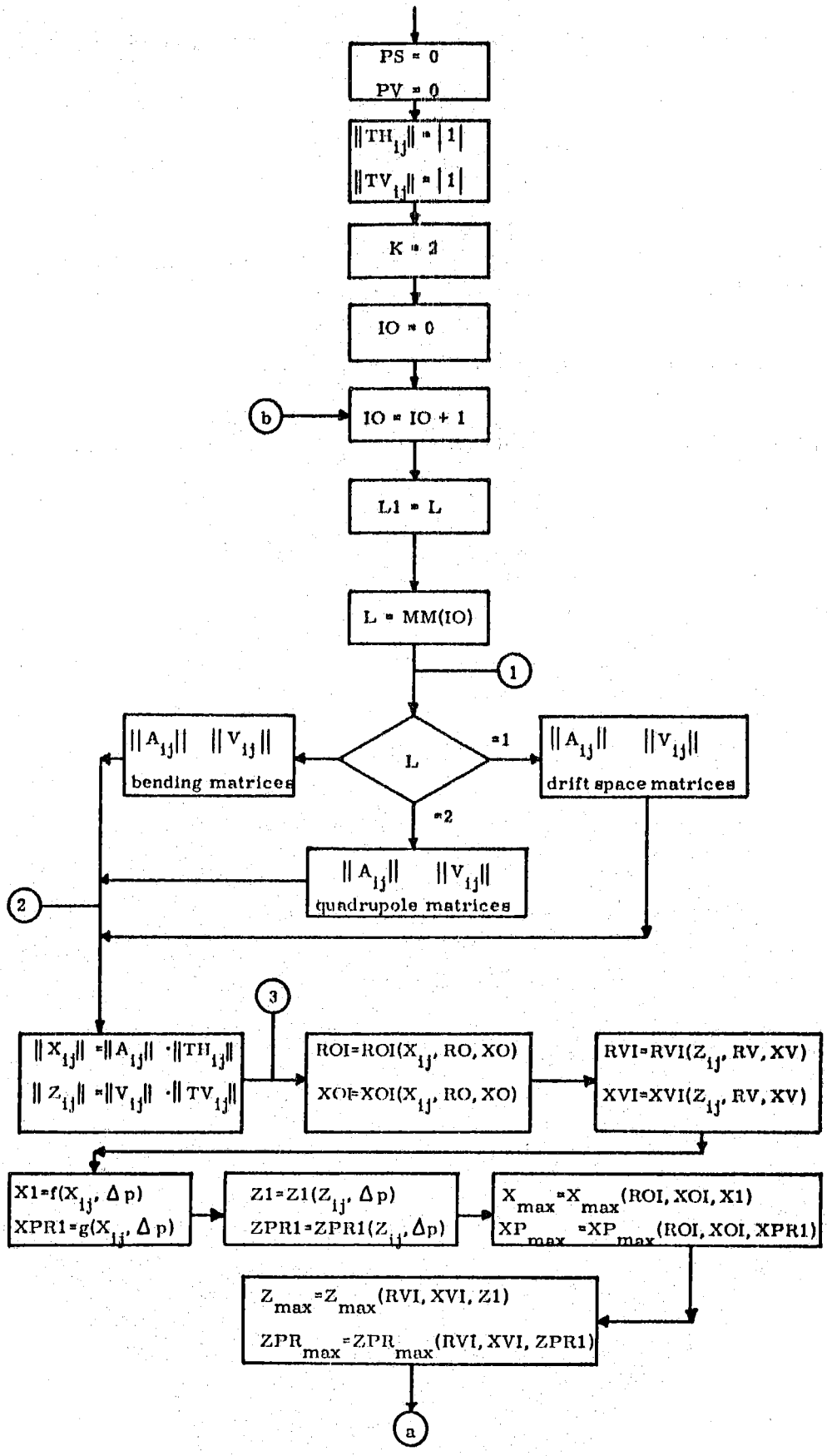
For each group of three lines and two columns the numbers represent the following quantities:

1^{st}	line	R	and	X	see relation (2)
2^{nd}	"	y	and	y'	see relation (6)
3^{rd}	"	y^{*}_{max}	and	y'^{*}_{max}	see relation (3')

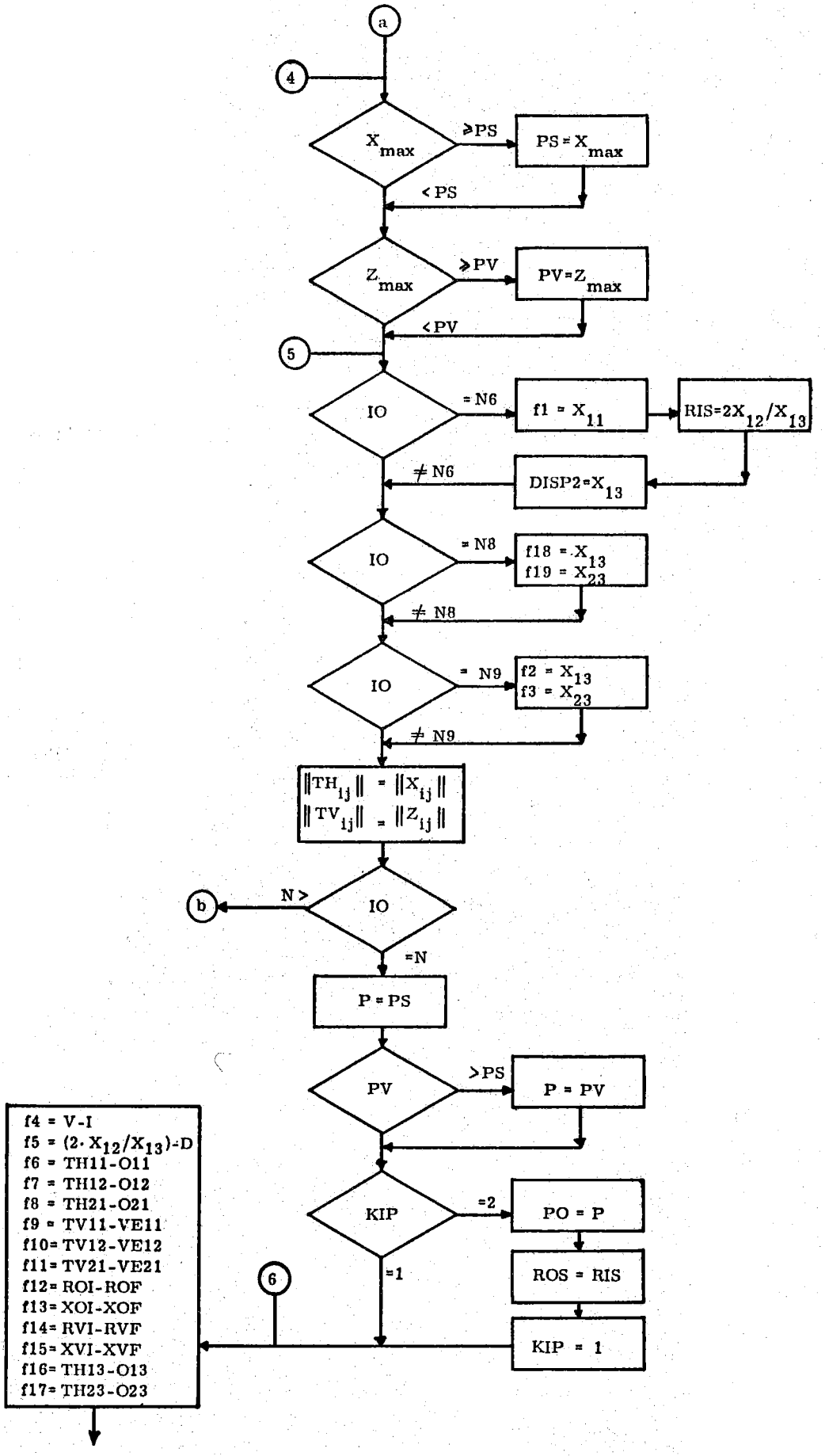
The values of R and X at the output of the last element, for the case of zero energy spread, must be equal for the values requested by equations 12, 13, 14 and 15.

In the numerical example showed in the following we have:

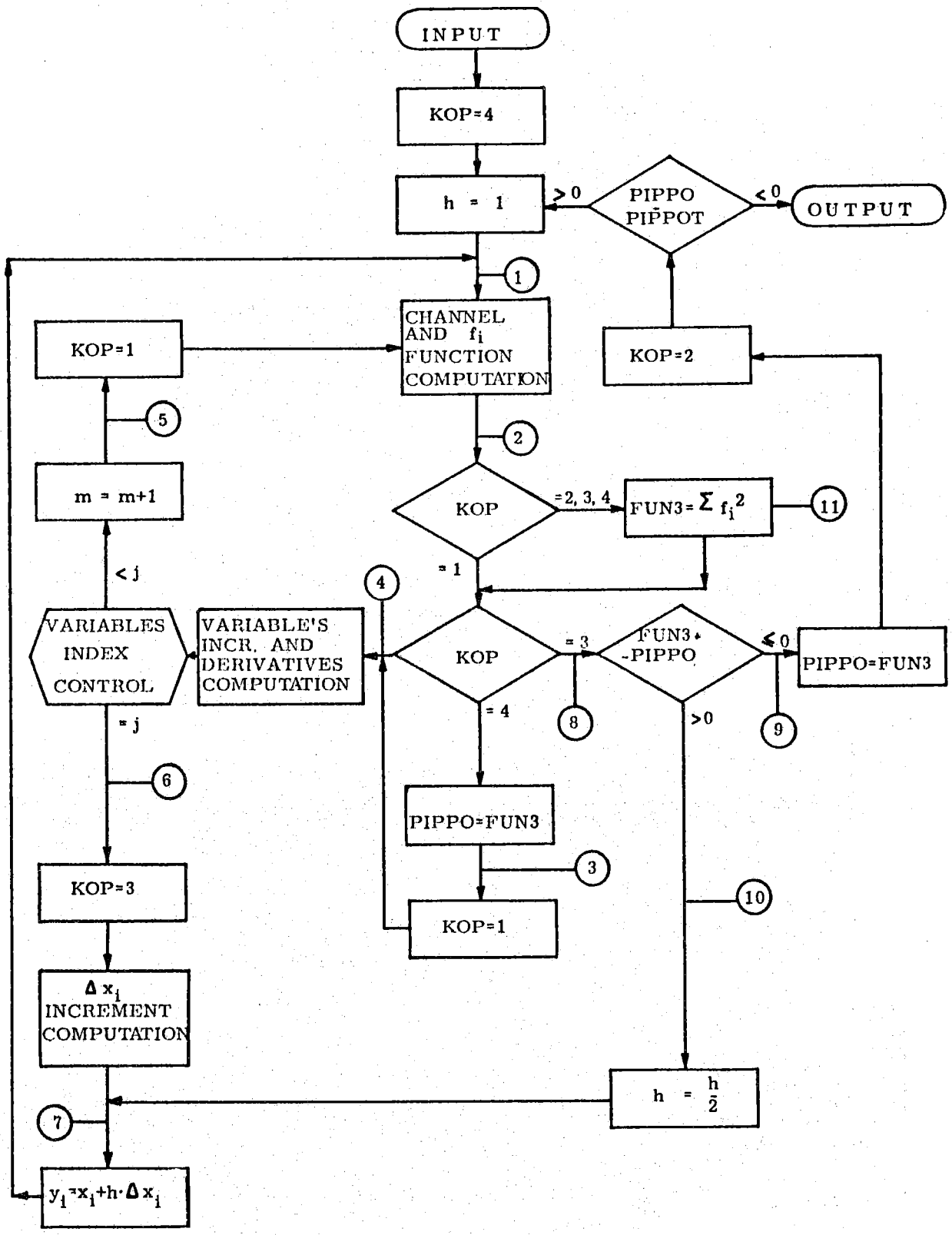
Orizontal plane	Requested value	Computed value
R	.8694462	.8694463
X	.6536449	.6536449
Vertical plane		
R	.1416358	.1476358
X	.8999918	.8999917



BLOCK DIAGRAM A



BLOCK DIAGRAM A



BLOCK DIAGRAM B

PARAMETRI DEL SISTEMA

```
 1   TRATTO DRITTO..... N.  1 ....  0.28333D 01
 2   QUADRUPOLO    ..... N.  1 ....  0.73000D 00  0.20000D 00
 3   TRATTO DRITTO..... N.  2 ....  0.10000D 01
 4   QUADRUPOLO    ..... N.  2 .... -0.73000D 00  0.20000D 00
 5   TRATTO DRITTO..... N.  3 ....  0.56667D 01
 6   QUADRUPOLO    ..... N.  3 ....  0.73000D 00  0.20000D 00
 7   TRATTO DRITTO..... N.  4 ....  0.10000D 01
 8   QUADRUPOLO    ..... N.  4 .... -0.73000D 00  0.20000D 00
 9   TRATTO DRITTO..... N.  5 ....  0.56667D 01
10   QUADRUPOLO    ..... N.  5 ....  0.73000D 00  0.20000D 00
11   TRATTO DRITTO..... N.  6 ....  0.10000D 01
12   QUADRUPOLO    ..... N.  6 .... -0.73000D 00  0.20000D 00
13   TRATTO DRITTO..... N.  7 ....  0.28333D 01
```

IL CANALE INIZIA ALLA SEZIONE 0   E TERMINA ALLA SEZIONE 13

```
                        EQUAZIONI IMPOSTE      VARIABILI

                        EQ(12)                 VAR( 1)
                        EQ(13)                 VAR( 2)
                        EQ(14)                 VAR( 3)
                        EQ(15)                 VAR( 4)

ORIZZONTALE   RI= 0.10000000D 02  XI= 0.          RF= 0.86944620D 00  XF= 0.65364490D 00
VERTICALE     RI= 0.10000000D 02  XI= 0.          RF= 0.14763580D 01  XF= 0.89999180D 00

 ERRORE INIZIALE= 0.25371700 02

   1 ITERAZIONE          ERRORE= 0.1561701D 01
   2 ITERAZIONE          ERRORE= 0.1372540D 00
   3 ITERAZIONE          ERRORE= 0.1367098D 00
   4 ITERAZIONE          ERRORE= 0.1362560D 00
   5 ITERAZIONE          ERRORE= 0.1355755D 00
   6 ITERAZIONE          ERRORE= 0.1351265D 00
   7 ITERAZIONE          ERRORE= 0.1350237D 00
   8 ITERAZIONE          ERRORE= 0.1350048D 00
   9 ITERAZIONE          ERRORE= 0.1349452D 00
  10 ITERAZIONE          ERRORE= 0.1349374D 00

NON CONVERGENZA

 F(12) = 0.13711875D 00      G( 1) = 0.48276365D 00
 F(13) = 0.13667546D 00      G( 2) =-0.62544570D 00
 F(14) = 0.30002957D 00      G( 3) = 0.70436017D 00
 F(15) = 0.85454130D-01      G( 4) =-0.69781965D 00

MAX.INV.ORIZZ. = 0.19493581D-01          MAX.INV.VERT. = 0.12198424D-01
```

```
                        EQUAZIONI IMPOSTE      VARIABILI

                        EQ(12)                 VAR( 2)
                        EQ(13)                 VAR( 3)
                        EQ(14)                 VAR( 4)
                        EQ(15)                 VAR( 5)

ORIZZONTALE   RI= 0.10000000D 02  XI= 0.          RF= 0.86944620D 00  XF= 0.65364490D 00
VERTICALE     RI= 0.10000000D 02  XI= 0.          RF= 0.14763580D 01  XF= 0.89999180D 00

 ERRORE INIZIALE= 0.1349374D 00

   1 ITERAZIONE          ERRORE= 0.1797517D-02
   2 ITERAZIONE          ERRORE= 0.2629265D-06
   3 ITERAZIONE          ERRORE= 0.3808116D-13

CONVERGENZA

 F(12) =-0.12144655D-07      G( 2) =-0.62687255D 00
 F(13) =-0.37217432D-07      G( 3) = 0.71784736D 00
 F(14) =-0.30540716D-07      G( 4) =-0.69520500D 00
 F(15) = 0.18872147D-06      G( 5) = 0.74349299D 00

MAX.INV.ORIZZ. = 0.19845683D-01          MAX.INV.VERT. = 0.12956628D-01
```

```
 G  1 = 0.4827636D 00      CA  1 = 0.1553647D 01      GRAD  1 = 0.4023030D 01
 G  2 =-0.6268726D 00      CA  2 =-0.1770413D 01      GRAD  2 = 0.5223938D 01
 G  3 = 0.7178474D 00      CA  3 = 0.1894528D 01      GRAD  3 = 0.5982061D 01
 G  4 =-0.6952050D 00      CA  4 =-0.1864410D 01      GRAD  4 = 0.5793375D 01
 G  5 = 0.7434930D 00      CA  5 = 0.1928073D 01      GRAD  5 = 0.6195775D 01
 G  6 =-0.7300000D 00      CA  6 =-0.1910497D 01      GRAD  6 = 0.6083333D 01
```

ULTIMI PARAMETRI DEL SISTEMA

```
 1   TRATTO DRITTO..... N.  1 ....  0.28333D 01
 2   QUADRUPOLO    ..... N.  1 ....  0.48276D 00   0.20000D 00
 3   TRATTO DRITTO..... N.  2 ....  0.10000D 01
 4   QUADRUPOLO    ..... N.  2 .... -0.62687D 00   0.20000D 00
 5   TRATTO DRITTO..... N.  3 ....  0.56667D 01
 6   QUADRUPOLO    ..... N.  3 ....  0.71785D 00   0.20000D 00
 7   TRATTO DRITTO..... N.  4 ....  0.10000D 01
 8   QUADRUPOLO    ..... N.  4 .... -0.69521D 00   0.20000D 00
 9   TRATTO DRITTO..... N.  5 ....  0.56667D 01
10   QUADRUPOLO    ..... N.  5 ....  0.74349D 00   0.20000D 00
11   TRATTO DRITTO..... N.  6 ....  0.10000D 01
12   QUADRUPOLO    ..... N.  6 .... -0.73000D 00   0.20000D 00
13   TRATTO DRITTO..... N.  7 ....  0.28333D 01
```

IL CANALE INIZIA ALLA SEZIONE 0   E TERMINA ALLA SEZIONE 13

```
                        EQUAZIONI IMPOSTE      VARIABILI

                        EQ(12)                 VAR( 1)
                        EQ(13)                 VAR( 2)
                        EQ(14)                 VAR( 3)
                        EQ(15)                 VAR( 4)
                        EQ( 4)                 VAR( 5)

ORIZZONTALE   RI= 0.10000000D 02  XI= 0.          RF= 0.86944620D 00  XF= 0.65364490D 00
VERTICALE     RI= 0.10000000D 02  XI= 0.          RF= 0.14763580D 01  XF= 0.89999180D 00

 ERRORE INIZIALE= 0.9846279D-06

   1 ITERAZIONE          ERRORE= 0.9845959D-06

NON CONVERGENZA

    F(12) = 0.91158270D-05      G( 1) = 0.48294061D 00
    F(13) =-0.26247213D-04      G( 2) =-0.62693050D 00
    F(14) =-0.21070993D-03      G( 3) = 0.71803714D 00
    F(15) = 0.11718373D-03      G( 4) =-0.69526348D 00
    F( 4) = 0.99225686D-03      G( 5) = 0.74349195D 00

  MAX.INV.ORIZZ. = 0.19845656D-01          MAX.INV.VERT. = 0.12956627D-01
```

```
                        EQUAZIONI IMPOSTE      VARIABILI

                        EQ(12)                 VAR( 2)
                        EQ(13)                 VAR( 3)
                        EQ(14)                 VAR( 4)
                        EQ(15)                 VAR( 5)
                        EQ( 4)                 VAR( 6)

ORIZZONTALE   RI= 0.10000000D 02  XI= 0.          RF= 0.86944620D 00  XF= 0.65364490D 00
VERTICALE     RI= 0.10000000D 02  XI= 0.          RF= 0.14763580D 01  XF= 0.89999180D 00

 ERRORE INIZIALE= 0.9845959D-06

   1 ITERAZIONE          ERRORE= 0.7837485D-06
   2 ITERAZIONE          ERRORE= 0.7426334D-06
   3 ITERAZIONE          ERRORE= 0.6028099D-06
   4 ITERAZIONE          ERRORE= 0.4485638D-06
   5 ITERAZIONE          ERRORE= 0.2998656D-06
   6 ITERAZIONE          ERRORE= 0.5898396D-07
   7 ITERAZIONE          ERRORE= 0.9380508D-14

CONVERGENZA

    F(12) = 0.74526191D-07      G( 2) =-0.62008722D 00
    F(13) =-0.25400708D-07      G( 3) = 0.72319953D 00
    F(14) = 0.23765063D-07      G( 4) =-0.69993895D 00
    F(15) =-0.51150318D-07      G( 5) = 0.74776141D 00
    F( 4) =-0.16061769D-09      G( 6) =-0.74902426D 00

  MAX.INV.ORIZZ. = 0.18853399D-01          MAX.INV.VERT. = 0.13079757D-01
```

```
 G  1 = 0.48294060 00      CA  1 = 0.15539310 01      GRAD  1 = 0.40245050 01
 G  2 =-0.62008720 00      CA  2 =-0.17608060 01      GRAD  2 = 0.51673940 01
 G  3 = 0.72319950 00      CA  3 = 0.19015780 01      GRAD  3 = 0.60266630 01
 G  4 =-0.69993900 00      CA  4 =-0.18707470 01      GRAD  4 = 0.58328250 01
 G  5 = 0.74776140 00      CA  5 = 0.19336000 01      GRAD  5 = 0.62313450 01
 G  6 =-0.74902430 00      CA  6 =-0.19352320 01      GRAD  6 = 0.62418690 01
```

ULTIMI PARAMETRI DEL SISTEMA

1	TRATTO DRITTO.....	N.	1	0.28333D 01	
2	QUADRUPOLO	N.	1	0.48294D 00	0.20000D 00
3	TRATTO DRITTO.....	N.	2	0.10000D 01	
4	QUADRUPOLO	N.	2	-0.62009D 00	0.20000D 00
5	TRATTO DRITTO.....	N.	3	0.56667D 01	
6	QUADRUPOLO	N.	3	0.72320D 00	0.20000D 00
7	TRATTO DRITTO.....	N.	4	0.10000D 01	
8	QUADRUPOLO	N.	4	-0.69994D 00	0.20000D 00
9	TRATTO DRITTO.....	N.	5	0.56667D 01	
10	QUADRUPOLO	N.	5	0.74776D 00	0.20000D 00
11	TRATTO DRITTO.....	N.	6	0.10000D 01	
12	QUADRUPOLO	N.	6	-0.74902D 00	0.20000D 00
13	TRATTO DRITTO.....	N.	7	0.28333D 01	

ERRORE INIZIALE= 0.9846275D-06

1	ITERAZIONE	ERRORE= 0.9012828D-06
2	ITERAZIONE	ERRORE= 0.7885543D-06
3	ITERAZIONE	ERRORE= 0.6820066D-06
4	ITERAZIONE	ERRORE= 0.6218849D-06
5	ITERAZIONE	ERRORE= 0.4835692D-06
6	ITERAZIONE	ERRORE= 0.3396842D-06
7	ITERAZIONE	ERRORE= 0.1905789D-06
8	ITERAZIONE	ERRORE= 0.2936805D-07
9	ITERAZIONE	ERRORE= 0.4744991D-13

CONVERGENZA

F(12) =-0.65486903D-07	G(2) =-0.61321343D 00
F(13) =-0.15508420D-06	G(3) = 0.72871176D 00
F(14) = 0.13821606D-06	G(4) =-0.70532144D 00
F(15) =-0.256460710-08	G(5) = 0.752270850 00
F(4) =-0.76449782D-10	G(6) =-0.769201950 00

MAX.INV.ORIZZ. = 0.17861115D-01 MAX.INV.VERT. = 0.13210448D-01

G 1 = 0.4829406D 00	CA 1 = 0.1553931D 01	GRAD 1 = 0.4024505D 01
G 2 =-0.6132134D 00	CA 2 =-0.1751019D 01	GRAD 2 = 0.5110112D 01
G 3 = 0.7287118D 00	CA 3 = 0.1908811D 01	GRAD 3 = 0.6072598D 01
G 4 =-0.7053214D 00	CA 4 =-0.1877926D 01	GRAD 4 = 0.5877679D 01
G 5 = 0.7522709D 00	CA 5 = 0.1939421D 01	GRAD 5 = 0.6268924D 01
G 6 =-0.7692019D 00	CA 6 =-0.1961125D 01	GRAD 6 = 0.6410016D 01

```
                    ULTIMI PARAMETRI DEL SISTEMA

 1   TRATTO DRITTO..... N.  1 ....  0.28333D 01
 2   QUADRUPOLO  ..... N.  1 ....  0.48294D 00  0.20000D 00
 3   TRATTO DRITTO..... N.  2 ....  0.10000D 01
 4   QUADRUPOLO  ..... N.  2 .... -0.61321D 00  0.20000D 00
 5   TRATTO DRITTO..... N.  3 ....  0.56667D 01
 6   QUADRUPOLO  ..... N.  3 ....  0.72871D 00  0.20000D 00
 7   TRATTO DRITTO..... N.  4 ....  0.10000D 01
 8   QUADRUPOLO  ..... N.  4 .... -0.70532D 00  0.20000D 00
 9   TRATTO DRITTO..... N.  5 ....  0.56667D 01
10   QUADRUPOLO  ..... N.  5 ....  0.75227D 00  0.20000D 00
11   TRATTO DRITTO..... N.  6 ....  0.10000D 01
12   QUADRUPOLO  ..... N.  6 .... -0.76920D 00  0.20000D 00
13   TRATTO DRITTO..... N.  7 ....  0.28333D 01


 ERRORE INIZIALE= 0.9846277D-06

   1 ITERAZIONE          ERRORE= 0.7811924D-06
   2 ITERAZIONE          ERRORE= 0.6561114D-06
   3 ITERAZIONE          ERRORE= 0.5668963D-06
   4 ITERAZIONE          ERRORE= 0.4986000D-06
   5 ITERAZIONE          ERRORE= 0.4355948D-06
   6 ITERAZIONE          ERRORE= 0.3740094D-06
   7 ITERAZIONE          ERRORE= 0.3428257D-06
   8 ITERAZIONE          ERRORE= 0.2743229D-06
   9 ITERAZIONE          ERRORE= 0.1948111D-06
  10 ITERAZIONE          ERRORE= 0.1195118D-06
  11 ITERAZIONE          ERRORE= 0.1833192D-07
  12 ITERAZIONE          ERRORE= 0.13586040-12

CONVERGENZA

 F(12) = 0.17537517D-06      G( 2) =-0.60629779D 00
 F(13) = 0.60570879D-07      G( 3) = 0.73467700D 00
 F(14) =-0.31065193D-06      G( 4) =-0.71169792D 00
 F(15) = 0.70217450D-07      G( 5) = 0.75712968D 00
 F( 4) = 0.17572301D-09      G( 6) =-0.79109199D 00

MAX.INV.ORIZZ. = 0.16868831D-01          MAX.INV.VERT. = 0.13352363D-01


 G  1 = 0.4829406D 00    CA  1 = 0.1553931D 01    GRAD  1 = 0.4024505D 01
 G  2 =-0.6062978D 00    CA  2 =-0.1741117D 01    GRAD  2 = 0.5052482D 01
 G  3 = 0.7346770D 00    CA  3 = 0.1916608D 01    GRAD  3 = 0.6122308D 01
 G  4 =-0.7116979D 00    CA  4 =-0.1886396D 01    GRAD  4 = 0.5930816D 01
 G  5 = 0.7571297D 00    CA  5 = 0.1945674D 01    GRAD  5 = 0.6309414D 01
 G  6 =-0.7910920D 00    CA  6 =-0.1988834D 01    GRAD  6 = 0.6592433D 01
```

ULTIMI PARAMETRI DEL SISTEMA

1	TRATTO DRITTO.....	N.	1	0.28333D 01	
2	QUADRUPOLO	N.	1	0.48294D 00	0.20000D 00
3	TRATTO DRITTO.....	N.	2	0.10000D 01	
4	QUADRUPOLO	N.	2	-0.60630D 00	0.20000D 00
5	TRATTO DRITTO.....	N.	3	0.56667D 01	
6	QUADRUPOLO	N.	3	0.73468D 00	0.20000D 00
7	TRATTO DRITTO.....	N.	4	0.10000D 01	
8	QUADRUPOLO	N.	4	-0.71170D 00	0.20000D 00
9	TRATTO DRITTO.....	N.	5	0.56667D 01	
10	QUADRUPOLO	N.	5	0.75713D 00	0.20000D 00
11	TRATTO DRITTO.....	N.	6	0.10000D 01	
12	QUADRUPOLO	N.	6	-0.79109D 00	0.20000D 00
13	TRATTO DRITTO.....	N.	7	0.28333D 01	

ERRORE INIZIALE= 0.9846283D-06

1	ITERAZIONE	ERRORE= 0.8077269D-06
2	ITERAZIONE	ERRORE= 0.7196390D-06
3	ITERAZIONE	ERRORE= 0.6693534D-06
4	ITERAZIONE	ERRORE= 0.6221049D-06
5	ITERAZIONE	ERRORE= 0.5592072D-06
6	ITERAZIONE	ERRORE= 0.5101308D-06
7	ITERAZIONE	ERRORE= 0.4474828D-06
8	ITERAZIONE	ERRORE= 0.3879381D-06
9	ITERAZIONE	ERRORE= 0.3505611D-06
10	ITERAZIONE	ERRORE= 0.2671559D-06
11	ITERAZIONE	ERRORE= 0.1876188D-06
12	ITERAZIONE	ERRORE= 0.1017779D-06
13	ITERAZIONE	ERRORE= 0.1157141D-07
14	ITERAZIONE	ERRORE= 0.1362811D-13

CONVERGENZA

F(12) = 0.83930009D-07	G(2) =-0.59932334D 00
F(13) =-0.21041769D-07	G(3) = 0.74129946D 00
F(14) = 0.55870765D-07	G(4) =-0.71960669D 00
F(15) =-0.54950392D-07	G(5) = 0.76255739D 00
F(4) =-0.13855817D-09	G(6) =-0.81580718D 00

MAX.INV.ORIZZ. = 0.15876546D-01 MAX.INV.VERT. = 0.13512780D-01

G 1 = 0.4829406D 00	CA 1 = 0.1553931D 01	GRAD 1 = 0.4024505D 01
G 2 =-0.5993233D 00	CA 2 =-0.1731074D 01	GRAD 2 = 0.4994361D 01
G 3 = 0.7412995D 00	CA 3 = 0.1925227D 01	GRAD 3 = 0.6177495D 01
G 4 =-0.7196067D 00	CA 4 =-0.1896848D 01	GRAD 4 = 0.5996722D 01
G 5 = 0.7625574D 00	CA 5 = 0.1952636D 01	GRAD 5 = 0.6354645D 01
G 6 =-0.8158072D 00	CA 6 =-0.2019662D 01	GRAD 6 = 0.6798393D 01

ULTIMI PARAMETRI DEL SISTEMA

1	TRATTO DRITTO.....	N.	1	0.28333D 01	
2	QUADRUPOLO	N.	1	0.48294D 00	0.20000D 00
3	TRATTO DRITTO.....	N.	2	0.10000D 01	
4	QUADRUPOLO	N.	2	-0.59932D 00	0.20000D 00
5	TRATTO DRITTO.....	N.	3	0.56667D 01	
6	QUADRUPOLO	N.	3	0.74130D 00	0.20000D 00
7	TRATTO DRITTO.....	N.	4	0.10000D 01	
8	QUADRUPOLO	N.	4	-0.71961D 00	0.20000D 00
9	TRATTO DRITTO.....	N.	5	0.56667D 01	
10	QUADRUPOLO	N.	5	0.76256D 00	0.20000D 00
11	TRATTO DRITTO.....	N.	6	0.10000D 01	
12	QUADRUPOLO	N.	6	-0.81581D 00	0.20000D 00
13	TRATTO DRITTO.....	N.	7	0.28333D 01	

INVILUPPO DEL SISTEMA • DELTA ENERGIA= 0.

0.10000000D 02	0.	0.10000000D 02	0.	
0.	0.	0.	0.	
0.10000000D-01	0.10000000D-02	0.10000000D-01	0.10000000D-02	1
0.10000000D 02	0.27333330 01	0.10000000 02	0.27333330 01	
0.	0.	0.	0.	
0.10366830-01	0.10000000-02	0.10366830-01	0.10000000-02	1
0.33561250 00	0.19734390 01	0.44048750 00	-0.20356940 01	
0.	0.	0.	-0.	
0.10926870-01	0.54585960-02	0.99238910-02	0.47646740-02	2
0.33561250 00	0.27734390 01	0.44048750 00	-0.12356940 01	
0.	0.	0.	-0.	
0.15249520-01	0.54585960-02	0.62505720-02	0.47646740-02	3
0.65431850 00	-0.38864900 01	0.26601140 01	-0.12787480 01	
0.	-0.	0.	-0.	
0.15407500-01	0.39093580-02	0.57226070-02	0.19388750-02	4
0.65431850 00	0.15801770 01	0.26601140 01	0.41879180 01	
-0.	-0.	-0.	-0.	
0.66861340-02	0.39093580-02	0.96194140-02	0.19388750-02	5
0.12060150 00	0.86294000 00	0.32773130 00	-0.16397060 01	
-0.	-0.	-0.	0.	
0.79342280-02	0.91059170-02	0.92366180-02	0.55238400-02	6
0.12060150 00	0.16629400 01	0.32773130 00	-0.83970580 00	
-0.	-0.	-0.	0.	
0.15182360-01	0.91059170-02	0.49791630-02	0.55238400-02	7
0.18719510 01	-0.66091660 01	0.12393920 01	-0.87377970 00	
-0.	0.	-0.	-0.	
0.15876550-01	0.23112810-02	0.43074500-02	0.28405060-02	8
0.18719510 01	-0.11424990 01	0.12393920 01	0.45928870 01	
-0.	0.	-0.	-0.	
0.50687740-02	0.23112810-02	0.13512780-01	0.28405060-02	9
0.85982400 00	0.12694920 01	0.17591850 00	-0.17193580 01	
-0.	-0.	-0.	0.	
0.52289320-02	0.34103210-02	0.13030830-01	0.75395300-02	10
0.85982400 00	0.20694920 01	0.17591850 00	-0.91935790 00	
-0.	-0.	-0.	0.	
0.76425390-02	0.34103210-02	0.70572830-02	0.75395300-02	11
0.86944630 00	-0.20796880 01	0.14763580 01	-0.18333420 01	
-0.	0.	-0.	0.	
0.76446030-02	0.33913970-02	0.61261750-02	0.26025800-02	12
0.86944630 00	0.65364490 00	0.14763580 01	0.89999170 00	
0.	0.	0.	0.	
0.36889740-02	0.33913970-02	0.44999950-02	0.26025800-02	13

INVILUPPO DEL SISTEMA * DELTA ENERGIA= 0.1000000D-02

0.1000000D 02	0.	0.1000000D 02	0.	
0.	0.	0.	0.	
0.1000000D-01	0.1000000D-02	0.1000000D-01	0.1000000D-02	1
0.1000000D 02	0.2733333D 01	0.1000000D 02	0.2733333D 01	
0.	0.	0.	0.	
0.1036683D-01	0.1000000D-02	0.1036683D-01	0.1000000D-02	1
0.3362399D 00	0.1975134D 01	0.4413679D 00	-0.2037738D 01	
0.	0.	0.	-0.	
0.1092636D-01	0.5453500D-02	0.9924384D-02	0.4759920D-02	2
0.3362399D 00	0.2775134D 01	0.4413679D 00	-0.1237738D 01	
0.	0.	0.	-0.	
0.1524488D-01	0.5453500D-02	0.6254907D-02	0.4759920D-02	3
0.6566132D 00	-0.3891946D 01	0.2668446D 01	-0.1278222D 01	
0.	-0.	0.	-0.	
0.1540304D-01	0.3902520D-02	0.5727767D-02	0.1935846D-02	4
0.6566132D 00	0.1574720D 01	0.2668446D 01	0.4188444D 01	
-0.	-0.	-0.	-0.	
0.6658214D-02	0.3902520D-02	0.9613901D-02	0.1935846D-02	5
0.1215515D 00	0.8626454D 00	0.3285608D 00	-0.1640739D 01	
-0.	-0.	-0.	0.	
0.7901712D-02	0.9070260D-02	0.9231441D-02	0.5516862D-02	6
0.1215515D 00	0.1662645D 01	0.3285608D 00	-0.8407394D 00	
-0.	-0.	-0.	0.	
0.1512087D-01	0.9070260D-02	0.4979851D-02	0.5516862D-02	7
0.1903721D 01	-0.6631872D 01	0.1242563D 01	-0.8738024D 00	
-0.	0.	-0.	-0.	
0.1581352D-01	0.2291914D-02	0.4309343D-02	0.2836879D-02	8
0.1903721D 01	-0.1165205D 01	0.1242563D 01	0.4592864D 01	
-0.	0.	-0.	-0.	
0.5115570D-02	0.2291914D-02	0.1349781D-01	0.2836879D-02	9
0.8498595D 00	0.1283385D 01	0.1767272D 00	-0.1721447D 01	
-0.	-0.	-0.	0.	
0.5280072D-02	0.3430255D-02	0.1301723D-01	0.7522259D-02	10
0.8498595D 00	0.2083385D 01	0.1767272D 00	-0.9214466D 00	
-0.	-0.	-0.	0.	
0.7718267D-02	0.3430255D-02	0.7057693D-02	0.7522259D-02	11
0.8576519D 00	-0.2091855D 01	0.1489321D 01	-0.1837861D 01	
-0.	0.	-0.	0.	
0.7719966D-02	0.3414636D-02	0.6129673D-02	0.2591229D-02	12
0.8576519D 00	0.6414785D 00	0.1489321D 01	0.8954722D 00	
0.	0.	0.	0.	
0.3657108D-02	0.3414636D-02	0.4503037D-02	0.2591229D-02	13

ERRORE INIZIALE= 0.9846276D-06

1	ITERAZIONE	ERRORE= 0.8709452D-06
2	ITERAZIONE	ERRORE= 0.8138344D-06
3	ITERAZIONE	ERRORE= 0.7618592D-06
4	ITERAZIONE	ERRORE= 0.7167358D-06
5	ITERAZIONE	ERRORE= 0.6732913D-06
6	ITERAZIONE	ERRORE= 0.6329801D-06
7	ITERAZIONE	ERRORE= 0.5968860D-06
8	ITERAZIONE	ERRORE= 0.5599995D-06
9	ITERAZIONE	ERRORE= 0.5326260D-06
10	ITERAZIONE	ERRORE= 0.5326136D-06
11	ITERAZIONE	ERRORE= 0.5326136D-06
12	ITERAZIONE	ERRORE= 0.5326136D-06

CONVERGENZA PARZIALE

F(12) =-0.36733251D-03	G(2) =-0.59596305D 00
F(13) =-0.34328111D-03	G(3) = 0.74480508D 00
F(14) =-0.23046714D-04	G(4) =-0.72420266D 00
F(15) = 0.10991927D-04	G(5) = 0.76547248D 00
F(4) = 0.52838110D-03	G(6) =-0.82922245D 00

MAX.INV.ORIZZ. = 0.15412643D-01 MAX.INV.VERT. = 0.13599951D-01

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